# Effect of Bracing Stiffness on Stability of Unbraced Frames

## Nazzal S. Armouti[1,*] & Bassel J. Hanayneh[2]

## Abstract

The effect of bracing stiffness on unbraced frames is evaluated. A single bay-one story frame having a span of 6 m and height of 4 m laterally supported by a spring (elastic support) at its joint is investigated. The frame stability is evaluated under various spring stiffness levels. It has been found that increasing lateral support stiffness increases the critical load to a point where spring stiffness reaches a threshold value to prevent the formation of the first buckling mode and triggers the second buckling mode without providing the frame with full lateral support. A two-bay two-floor frame laterally braced by shear wall is also investigated. It has been also found that providing unbraced frames with lateral supports with enough stiffness can convert the sway mode of buckling to non-sway mode of buckling which is a big advantage in the design of steel frames.

**Keywords:** Stability of steel frames; unbraced frames; bracing stiffness.

## 1. Introduction

Stability of steel frames depends mainly on its stiffness and bracing. In general, first mode of buckling can be prevented by properly bracing the member at designated points. Various types of bracing for high rise buildings are examined for optimization, (Haque et al. 2018), (Metre et al. 2017), (Siddiqi et al. 2014).

Bracing stiffness plays a major role in the selection of bracing type, for example (Green et al. 2004) examined the requirements of stiffness for axially loaded lipped cee studs, (Klasson et al. 2019) examined the requirements of stiffness for timber structures, while (Mutton et al. 1973) studied the effect of end restraints on the stability of I-beams. It is well established that the first buckling mode of a simply supported column (Euler column) can be prevented if it is supported at mid-span with elastic spring with specified minimum stiffness (Timoshenko and Gere, 1961). Such minimum stiffness will also trigger the formation of the second buckling mode. This problem is presented in Timoshenko and Gere for a single span column supported with hinge and roller as shown in Figure 1. This figure also shows the first two buckling loads and their mode shapes.
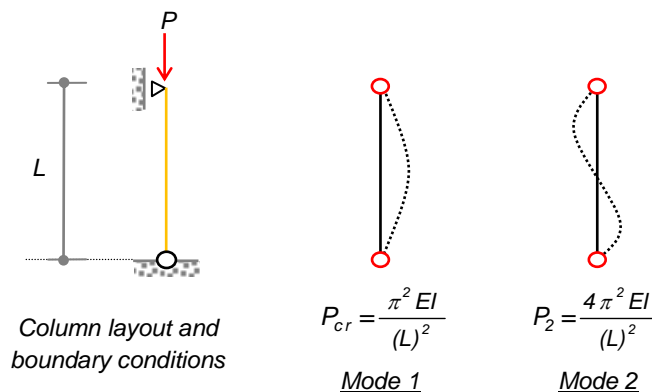


$$P_{cr} = \frac{\pi^2 EI}{(L)^2}$$

Mode 1

$$P_2 = \frac{4\pi^2 EI}{(L)^2}$$

Mode 2

**Figure 1. First two buckling modes of simply supported column**

[1] Professor, Department of Civil Engineering, University of Jordan, Amman 11942, Jordan * (Corresponding author: Email: armouti@ju.edu.jo). Mobile: +962 7 99 47 66 99
[2] Associate Professor, Department of Civil Engineering, University of Jordan, Jordan

If this column is supported at mid-span by elastic spring with constant, $\alpha$, as shown in Figure 2, Timoshenko and Gere have shown that the first buckling mode takes place at, $\alpha = 0$, whereas the second mode takes place at a minimum value of, $\alpha$, equals to

$$\alpha_{min} = 16 \, \frac{P_e}{L}$$

where

$\alpha$ = Spring constant (stiffness quantity, kN /m)

$P_e$ = Euler buckling load, $P_e = \dfrac{\pi^2 EI}{L^2}$
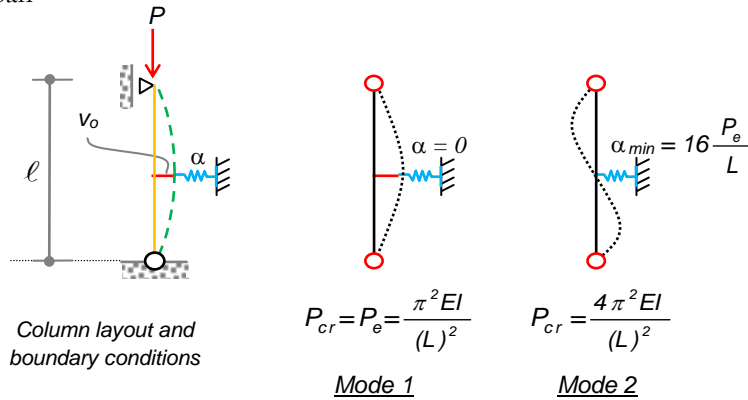
$L$ = Column span



Figure 2. First two buckling modes of simply supported column with spring support at midspan

It has also been shown by Timoshenko and Gere that for values of, $\alpha$, between, 0 and $\alpha_{min}$, the critical load marks transition values between the first mode and the second mode in nearly straight line relationship as shown in Figure 3.
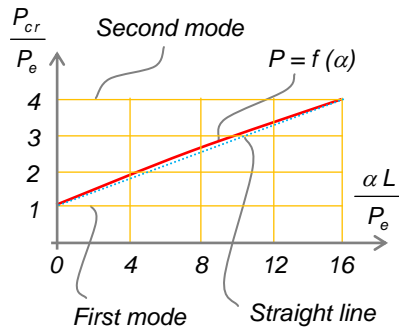


Figure 3. Variation of single column buckling load as function of spring stiffness

Figure 3 shows a non-dimensional relationship between the buckling load, $P_{cr}$, and the constant, $\alpha$, between the first mode and second mode of buckling. It can be observed from this figure that the first mode takes place at, $\alpha = 0$, whereas the second mode takes place at value of, $\alpha L/P_e = 16$. Note that the buckling load increases with the increase of, $\alpha$, in the first mode until, $\alpha$, reaches an, $\alpha_{min}$, which triggers the second mode.

Even though the solution of this problem for a single span column is mathematically manageable, the treatment of more complex structures becomes tedious and mathematically unmanageable. Consequently, revert to finite element method, FEM, deems necessary.

## 2. Finite Element Modeling and Verification

The exact critical load of Euler column presented in the previous section is well established as mentioned previously. In order to introduce finite element method in buckling analysis and to verify its validity and accuracy for this matter, the previous column solution will be obtained by finite element method in the next sections. In this case, the critical load is found by utilization of geometric stiffness matrix method which, in fact, is a form of Finite Element Analysis (Chen and Lui, 1987). If the right hand rule sign convention for forces and moments is used, the element stiffness matrix, $[k_m]$, and the element geometric stiffness matrix, $[k_{Gm}]$, take the following form

$$[k_m] = \frac{EI}{L^3} \begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}, \qquad [k_{Gm}] = \frac{P}{30L} \begin{bmatrix} 36 & 3L & -36 & 3L \\ 3L & 4L^2 & -3L & -L^2 \\ -36 & -3L & 36 & -3L \\ 3L & -L^2 & -3L & 4L^2 \end{bmatrix}$$

where

E = Young modulus
I = moment of inertial of the section
L = length of the element
P = externally applied load

As well established, assembly of the global stiffness and geometric stiffness matrices, followed by standard solution of eigenvalue problem result in obtaining the buckling loads and their buckling shapes of the system.

In order to examine the effectiveness of FEM in obtaining the results presented in the previous section, a single span column is selected with dimensions and properties as shown in Figure 4. The dimensions of, L = 10 m, and flexure stiffness, EI = 10,000 kN.m², are used.
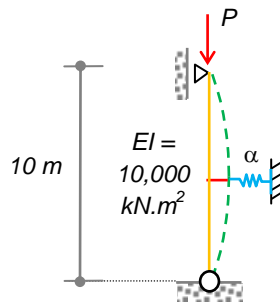


Figure 4. Column layout and boundary conditions

Figure 5 shows a 2-element FEM model analyzed with software program prepared for this task to produce the results shown in the same figure. It can be seen that the solution is very accurate for the case without spring, and is overestimated by, 5%, for the case with spring.



$$P_{cr} = \frac{\pi^2 EI}{(L)^2} \qquad P_{cr} = \frac{4\pi^2 EI}{(L)^2}$$

$$\alpha_{min} = 16\frac{P_e}{L}$$
$$= 1,579.137$$

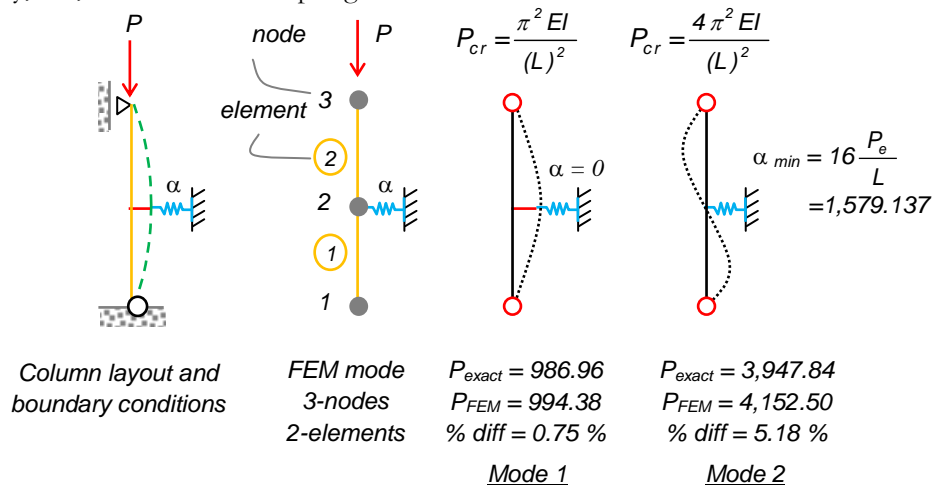| Column layout and boundary conditions | FEM mode 3-nodes 2-elements | $P_{exact}$ = 986.96 $P_{FEM}$ = 994.38 % diff = 0.75 % | $P_{exact}$ = 3,947.84 $P_{FEM}$ = 4,152.50 % diff = 5.18 % |
| --- | --- | --- | --- |
|  |  | *Mode 1* | *Mode 2* |

Figure 5. Two-element FEM model for single span column with spring support at midspan

In order to improve the results, a finer mesh is selected with, 4-elements, FEM model as shown in Figure 6. It can be seen that the solution is very accurate for the case without spring, 0.05% error, and is also very accurate with spring with an overestimated error of, 0.25%.



$$P_{cr} = \frac{\pi^2 EI}{(L)^2}$$

$$P_{cr} = \frac{4\pi^2 EI}{(L)^2}$$

$$\alpha_{min} = 16\frac{P_e}{L} = 1{,}579.137$$

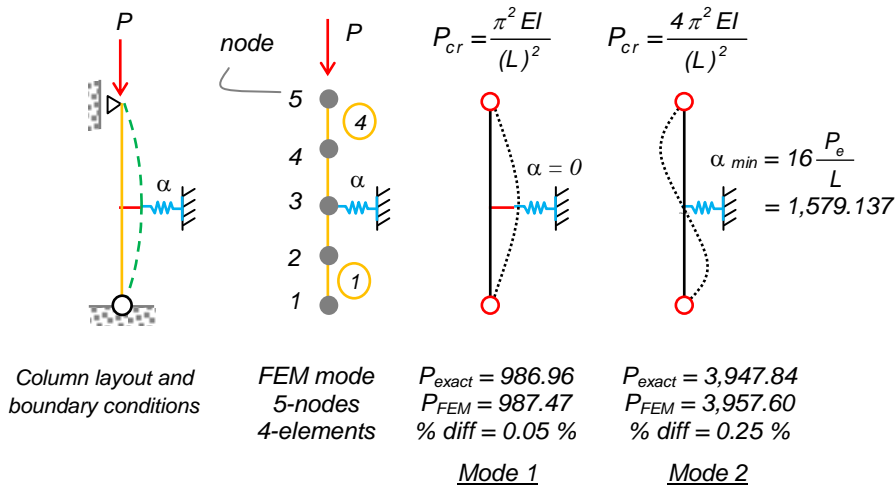| Column layout and boundary conditions | FEM mode 5-nodes 4-elements | $P_{exact} = 986.96$ $P_{FEM} = 987.47$ % diff = 0.05 % | $P_{exact} = 3{,}947.84$ $P_{FEM} = 3{,}957.60$ % diff = 0.25 % |
|---|---|---|---|
| | | _Mode 1_ | _Mode 2_ |

Figure 6. Four-element FEM model for single span column with spring support at midspan

Consequently, it would be fair to say that a column with four elements yields acceptable and very accurate FEM model for all engineering purposes.

## 3. Frame Case Model

The frame selected for this study consists of one-bay one-floor frame with dimensions as shown in Figure 7. The frame is supported with two hinges at, A and B, and is subjected to two vertical axial loads at joints C and D. The frame is also laterally supported by a spring (flexible support) at joint D with spring constant (stiffness), $\alpha$, in order to explore the effect of variation of lateral stiffness on the stability of the frame. This model may be expanded to more bays and levels as desired in practice.
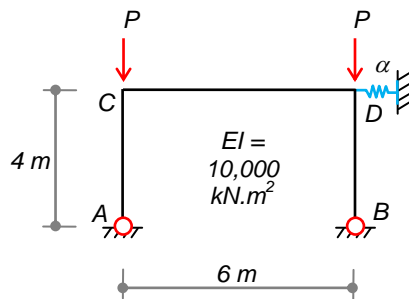


Figure 7. Frame layout

For the purpose of performing this analysis, the frame is assumed to be fully supported in the transverse direction, and hence the frame buckles in its plane. The active moment of inertia in the plane of the frame is selected arbitrary as, $EI = 10{,}000$ kN.m².

Figure 8 shows the modeling of the frame utilized in this study which is discretized into, 10, nodes and, 9, elements. The spring is introduced at the frame top corner with stiffness coefficient, $\alpha$.
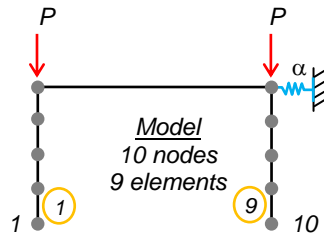
Figure 8. Frame discretization

## 4. Analysis and Discussion

Using the model described in the previous section, stability analysis is performed for various cases of interest. The buckling of the frame without spring ($\alpha = 0$) is obtained as shown in Figure 9 which shows the first two buckling loads and their buckling shapes.



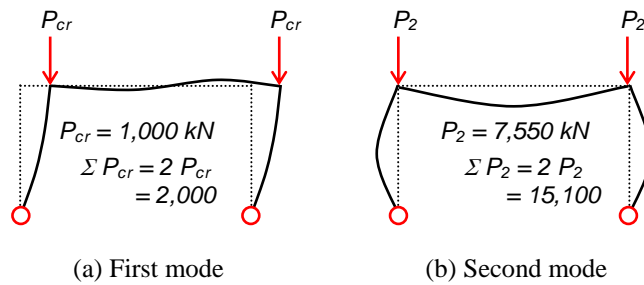(a) First mode                                      (b) Second mode

Figure 9.  First two buckling loads and modes of frame without
spring, $\alpha = 0$

Figure 9(a) shows the first buckling load, $P_1 = 1,000$ kN, with its buckling shape which is of the sway (unbraced) type as shown in the same figure.  Note that, $P_1$, is the critical load, $P_{cr}$, as it is the smallest of all buckling loads (smallest eigenvalule).  Figure 9(b) shows the second buckling load, $P_2 = 7,550$ kN, with its buckling shape which is of the non-sway (braced) type as shown in the same figure.

In order to find the minimum spring stiffness that triggers the second mode, and analogous to the case of the single column presented earlier, the analysis proceeds by increasing the constant, $\alpha$, incrementally until the second mode is triggered and becomes the first mode.  By doing so, the minimum, $\alpha_{min}$, required to trigger the second mode is found to be, $\alpha = 4,400$ kN/m, which also makes, $P_2 = P_{cr}$, as shown in Figure 10(b). It should be pointed out that any value of, $\alpha$, more than zero and less than the specified minimum will increase the buckling load and at the same time maintains the first buckling shape as shown in Figure 10(a).
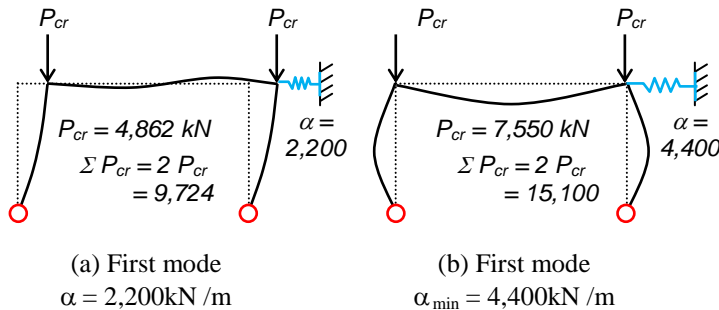


(a) First mode                                      (b) First mode
$\alpha = 2,200$kN /m                              $\alpha_{min} = 4,400$kN /m

Figure 10. Variation of the first buckling load and mode of
frame for various values with spring, $\alpha$

Analogous to the case of the single column, if the lowest buckling load of the frame without spring is taken as a reference quantity, $P_{ek}$, then the results above can be summarized in terms of, $P_{ek}$, as in Table 1

**Table 1 Summary of frame buckling analysis ($P_{ek}$ = 2,000 kN)**

|  | $\alpha = 0$ | | $\alpha = 4,400$ kN /m | |
|---|---|---|---|---|
|  | Mode load | Mode type | Mode load | Mode type |
| $P_{ek}$ | 2,000 | | 2,000 | |
| $P_1 = P_{cr}$ | 2,000 | sway | 15,100 | non-sway |
| $P_2$ | 15,100 | non-sway | irrelevant | irrelevant |
| $P_{cr} / P_{ek}$ | 1 | | 7.55 | |

Analogous to the case of single column, the variation of the critical load, $P_{cr}$, as function of the spring stiffness, $\alpha$, is calculated and expressed in non-dimensional quantities as follows

The abscissa:       $\dfrac{\alpha L}{P_{ek}} = \dfrac{\alpha (4)}{2,000} = \dfrac{\alpha}{500}$       $\rightarrow$       $\alpha = 500 \left( \dfrac{\alpha L}{P_{ek}} \right)$

The ordinate:       $\dfrac{P_{cr}}{P_{ek}} = \dfrac{P_{cr}}{2,000}$

The results of the above expressions are summarized in Table 2 and shown in Figure 11.

**Table 2 Variation of frame buckling with variation of spring stiffness**

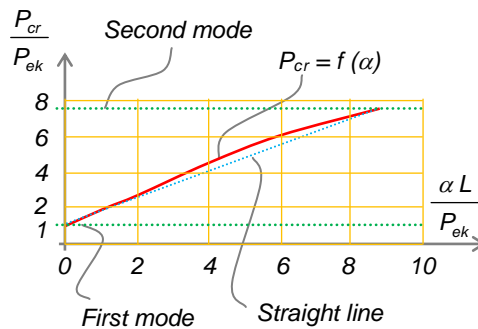| $\alpha L/P_{ek}$ | 0 | 2 | 4 | 6 | 8 | 8.8 |
|---|---|---|---|---|---|---|
| $\alpha = 500$ x $(\alpha L/P_{ek})$ | 0 | 1,000 | 2,000 | 3,000 | 4,000 | 4,400 |
| $P_1 = P_{cr}$ (FEM) | 2,000 | 5,640 | 9,076 | 12,112 | 14,426 | 15,100 |
| $P_{cr} /P_{ek}$ | 1 | 2.82 | 4.54 | 6.06 | 7.21 | 7.55 |



Figure 11. Variation of frame buckling load
as function of spring stiffness

Table 2 and Figure 11 show that the frame follows basically the same trend in behavior of a single column buckling, meaning, the buckling load increases with increasing the stiffness of the lateral support almost in linear relationship.  The buckling load keeps increasing with the original first mode of buckling until a threshold value of stiffness, $\alpha_{min}$, triggers the second mode to be the first critical mode. It can be also observed that when this threshold value of, $\alpha_{min}$, is reached, the buckling mode shifts from sway mode to non-sway mode.

This observation is very important which indicates that sway frames which usually result in large amplification factors for displacements and moments due to second order effect (P-Δ effect) can be converted into non-sway frames which usually experience less severe amplifications in displacement and moments due to second order effect.

## 5. Application in Practice

As pointed out in the previous section, lateral bracing of frames may convert sway frames to non-sway frame type. As this fact is important in practice, especially in steel frames, a sway frame can be converted into non-sway frame by introducing some form of elastic support in the lateral direction. The question usually arises on the amount of stiffness of lateral support required to achieve this conversion, (ACI 2014), (AISC 2017). In general, shear walls can be used to achieve this issue as they are very stiff and can serve as lateral flexible support, and hence are selected as sample solution to this problem in this study.

To illustrate the utilization of this concept, a simple example is presented herein which consists of two-bay two-floor framing system as shown in Figure 12. The frame is considered for this analysis by extending the geometry and properties used in the previous section, namely, 6m x 4m, bays with flexure stiffness for the frame members, $EI_{fr}$ = 10,000 kN /m², for all members as shown in Figure 12(a).



(a) Frame layout

(b) Frame buckling load and buckling shape

$P_{cr} = 2,037\ kN$

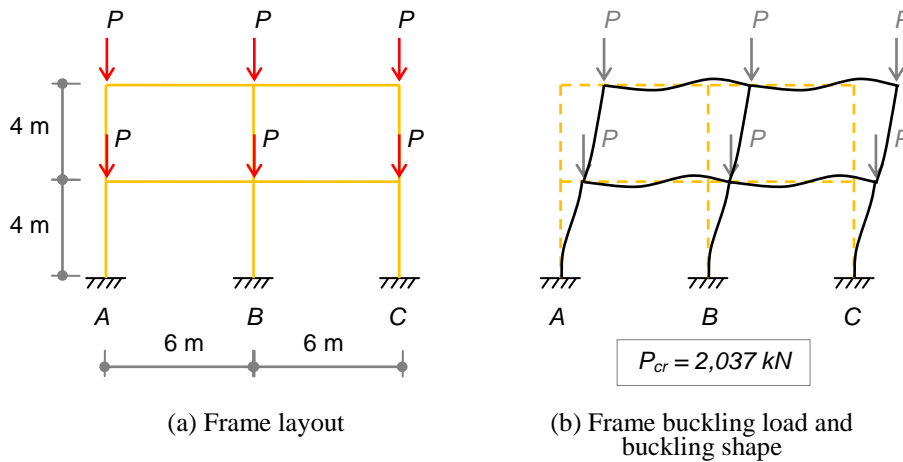Figure 12. Two-bay two-floor frame layout
EI = 10,000 kN.m² (all members)

Performing buckling analysis for this frame using FEM results in a critical buckling load, $P_{cr}$ = 2,037 kN, and a sway buckling shape as shown in Figure 12(b).

In order to convert this sway buckling mode to non-sway buckling mode, a shear wall is added to laterally support this frame as shown in Figure 13. The buckling analysis of the frame and the shear wall combined system can be performed using FEM as has done earlier.
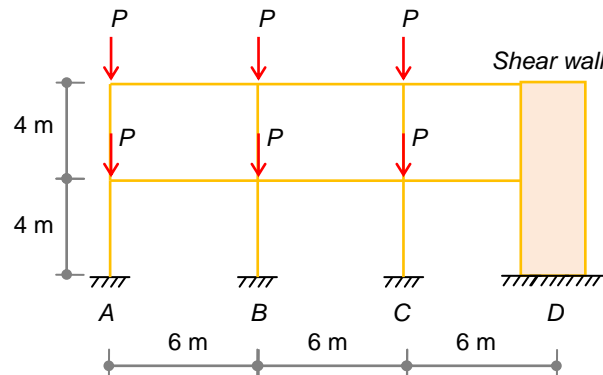


Figure 13. Frame and shear wall arrangement
EI = 10,000 kN.m² (all members)

The analysis proceeds by assigning a flexure stiffness for the shear wall, $EI_{sw}$, equals to the frame stiffness, i.e. $EI_{sw}$ = $EI_{fr}$ = 10,000 kN.m². In this case, the resulting buckling load and buckling shape are as shown in Figure 14. Note that the critical load increases due to the presence of the shear wall. It should be pointed out that this case represents a frame with additional bay due to the small stiffness value of the shear wall and hence, the buckling shape maintains the original sway mode as it took place before since the shear wall behaves as another column in this case.
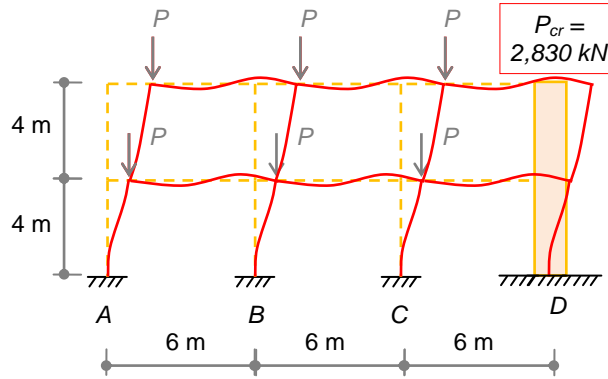
Figure 14. Frame and shear wall first buckling mode
EI = 10,000 kN.m² (all members)

The next step in this analysis proceeds by incrementally increasing the stiffness of the shear wall until the non-sway mode takes place. After incremental increase of the wall flexure stiffness, it has been found that the flexure stiffness that converts the sway mode of the frame to non-sway mode is in the neighborhood of , $EI_{sw}$ = 2,000,000 kN.m², which amounts to, 200 times, the value of, $EI_{fr}$, of the frame members.

Figure 15 shows the state of buckling at this stage, which indicates that the non-sway mode takes over with a buckling load, $P_{cr}$ = 8,082 kN. The critical load amounts to, 3.97, times the critical load of the frame without shear wall. These results are summarized in Table 3 for clarity.
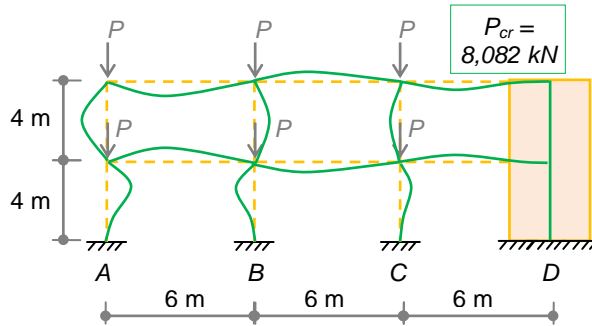


Figure 15. Frame and shear wall first buckling mode
Shear wall, EI = 2 x10⁶ kN.m²

**Table 3 Summary of buckling loads and shapes for frame-shear wall system**

|  | Frame without shear wall | Frame with shear wall | | |
|---|---|---|---|---|
|  | Frame stiffness | Shear wall stiffness | Shear wall stiffness | Shear wall stiffness |
|  | 10,000 | 0 | 10,000 | 2,000,000 |
| $EI_{sw}$ /$EI_{fr}$ | - | 0 | 1 | 200 |
| $P_{cr}$ | 2,037 | 2,037 | 2,830 | 8,082 |
| $P_{cr}$ / $P_{ek}$ | 1 | 1 | 1.39 | 3.97 |
| Mode shape | Sway | Sway | Sway | non-sway |

Since the flexure stiffness is cubically proportional to depth, the above results also imply that the ratio of the shear wall section depth to frames section depth equals to

$$\frac{EI_{sw}}{EI_{fr}} = 200 = \left(\frac{h_{sw}}{h_{fr}}\right)^3 \rightarrow \left(\frac{h_{sw}}{h_{fr}}\right) = (200)^{1/3} = 5.85$$

The above result indicates that a shear wall of depth equals to, 5.85, times the depth of the frame member should be enough to convert the sway mode of buckling of the frame to non-sway buckling mode.

## 6. Conclusions

The effect of bracing stiffness on global stability of unbraced frames is an important issue especially in steel frames.  Bracing is required in general, not only in global stability of frames.  Contrary to many beliefs, this study reasserts the fact that stiffness and not strength controls the required bracing systems. Review of the behavior of single column under flexible support (stiffness) is reviewed as presented and proven in the literature that the first mode of buckling can be prevented if minimum amount of stiffness is provided independent of the strength of the support which also implies that full support is not really necessary to prevent the first mode of buckling. Extension of this concept to stability of frames is presented in this study by investigating one-frame supported by lateral spring at its upper joint where it is shown that a stiffness threshold value is required to prevent the first buckling mode and to trigger the second mode without fully supporting the joint of the frame.  It is shown that the behavior of frames follows basically the same trend of behavior of single column. It is also shown in this study that an unbraced frame can be converted into a braced frame by supporting the frame with lateral stiffness without fully supporting the frame. As this concept is important in practice, the study also examined the bracing of frames by stiff members such as shear walls.  It is shown that this concept works if the frame is supported by flexible member, a shear wall in this case, and it is feasible to do so in practice.

## References

ACI. (2019). Building Code Requirements for Structural Concrete (318M-14). American Concrete Institute, Farmington Hills, MI 48331.

AISC. (2017). Specification for Structural Steel Buildings. American Institute of Steel Construction, Chicago, IL 60601.

Chen, W. F., & Lui, E. M. (1987). Structural Stability, Theory and Implementation. New York: Elsvier.

Green, P. S., Sputo, T., & Urala, V. (2004). Bracing Strength and Stiffness Requirements for Axially Loaded Lipped Cee Studs. Proceedings of the 17th International Specialty Conference on Cold Formed Steel Structures, November4-5. pp. 511-530, Orlando, Florida, USA.

Haque, A., Masum, A. Ratul, M., and Tafheem, Z. (2018). Effect of Different Bracing Systems on the Structural Performance of Steel Building. Proceeding of the 4th International Conference on Civil Engineering for Sustainable Development, February 9-11, Paper No. ICCESD-2018-4905, pp. 1 -8,Khulna, Bangladesh.

Klasson, A., & Crocetti, R. (2019).The Effect of the Bracing Stiffness of Timber Structures of the Stiffness of Its Members. Structures, 19, 41-47.

Metre, S., Ghule, S., C., & Kiran, R. (2017). Comparative Study of Different Types of Bracing Systems by Placing at Different Locations. International Research Journal of Engineering and Technology (IRJET),04(8), 2023 - 2027.

Mutton, B. C., & Trahair, N. S. (1973). Stiffness Requirements for Lateral Bracing. Journal of the Structural Division, 99(10), 2167 - 2182.

Siddiqi, Z. A., Hameed, R., & Akma, U. (2014). Comparison of Different Bracing Systems for Tall Buildings. Pak. J. Engg. Sci., 14, 17-26.

Timoshenko, S. P., & Gere J. M. (1961). Theory of elastic stability. New York: McGraw-Hill.