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Widening the Spirit of an Architect: The Geometrical Problem of Inserting a Perspective Drawing In a Photo

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Abstract

Geometry is a necessary purveyance in an architect's education. Its role is not confined in the correct drawing of various objects; rather it focuses in the broadening of the imagination and the perception of space, necessary instruments in architectural design. One of the objects of geometrical education is perspective drawing, which is a principal means of depicting both the constructed and the drawn recommendation of the architectural space. Descriptive geometry and Perspective set the rules of the perspective drawing. In many instances, though, it is necessary to reverse this process and use some helpful measurements in the architectural depiction of an object under inquiry, that have occurred from a perspective drawing or a photo of a building. In this paper I discuss, firstly, these reverse geometrical processes and then I expand on the problem of incorporating a perspective drawing or a three dimensional building model in a photo of an actual space, so as to look as the building in drawing has the same vanishing points as the photo. The various design software enable the perspective drawing but they don't elaborate on the geometrical process of perspective vision. This paper, through its theme of incorporating a perspective drawing in a photo, essentially suggests the cooperation of analog geometrical methods and computers, highlighting at the same moment the necessity of geometrical education for architects in parallel with the training in the operation of design software, for an enhanced view of space.

Keywords: Perspective; geometry; vanishing point; horizon line

1. Introduction

In this paper I discuss the subject of incorporating a perspective drawing in a photo. Given the knowledge of setting up a perspective drawing with geometrical methods, here are presented the geometrical procedures for the finding of the necessary elements, in order to photograph a given space in such a way so as to be able to incorporate the perspective drawing inside the photo, with the aid of some image changing software, like Photoshop. The photo of the space and the constructed perspective drawing will look like they have the same point of view and the same vanishing points. Therefore the perspective drawing will look like it was a part of the space during the taking of the photo.

The purpose of this paper is to show the importance of a geometrical education in an architect's training both as an asset for the depiction and representation of space and as a means to broaden one's imagination and cultivate his perception of space. In the discussion of this particular geometrical problem we can also see the importance of a geometrical perception of space, which is mostly cultivated through traditional techniques, and its relation to the new technological methods of representation.

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1. Basic concepts of perspective drawing

For the construction of a perspective drawing we assume that the observer is fixed in a certain point in space and observes an object with one eye or through a camera lens. What supposedly happens is that the optical beams which connect the eye with the object meet a vertical plane at some points, which, if connected in the correct way, form the image of perspective drawing (Image 1).

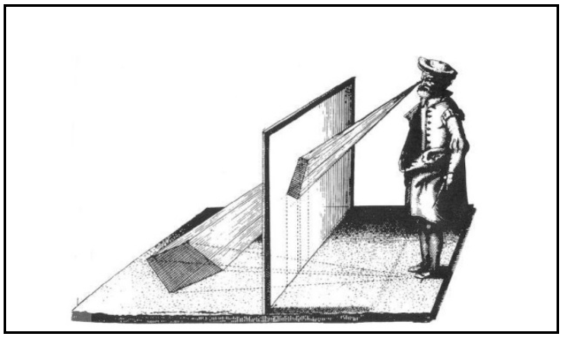

Image 1: The optical beams, engraving, Hendrik Hondius, Marolois 1614.

It is called **perspective image**, the central projection of an object on a surface. The centre of projection **O** is called **point of view** or **visual centre** (Image 2).

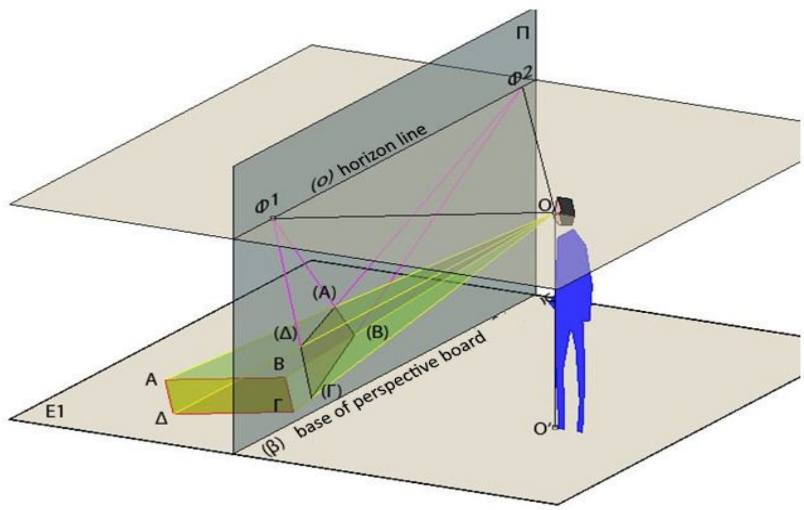

Image 2 : The perspective image of an object as the central projection of the object on the perspective board.

A projection surface can be any surface S. In this paper we will use as a projection surface a **vertical plane** π, called perspective **board**. The perspective board meets the horizontal plane of the ground $\mathbf{E_1}$ across the straight line β, which is called the base of the perspective board (Image 2). The straight lines that project the object from the point of view O on the perspective board π are called **optical beams**. The horizontal plane that passes through the point of view O is the **horizon** level, which meets the perspective board π in a straight line o, parallel to β, the **horizon line**. The distance of the horizon line from the base of the perspective board suggests the height of the point of view. The optical beams ΟΑ, ΟΒ, ΟΓ… meet the perspective board π at the points (Α), (Β), (Γ)… respectively, determining thus the perspective images of the apices of the figure.

The surface of the ground is usually considered as a reference surface, to which the horizontal projections of the figures in space are referred to. The vertical plane of the perspective board is reposed on the horizontal plane, rotating round β, so as the horizontal projections of the figures and their perspective images are at the same drawing plane.

2. The example of the perspective drawing of a horizontal square

The square $AB\Gamma\Delta$ of Image 3 belongs in the horizontal plane E_1. The sides of $AB\Gamma\Delta$, if extended in the appropriate way, meet the base β of the perspective board at the points 1, 2, 3 and 4, which coincide with their perspective images. The vanishing point of the side AB of the square is the point Φ_1 of the horizon line. The parallel to AB that comes through the projection O' of the vision point O, determines on the perspective board the projection $\Phi_1'$of the vanishing point Φ_1 of the line AB, from which Φ_1 occurs on the horizon line. The line $\Gamma\Delta$, being parallel to AB, as well as any other parallel to them, will have the same vanishing point Φ_1. Respectively, the vanishing point of the second pair of the parallel sides $B\Gamma$ and $A\Delta$ of the square is the point Φ_2 of the horizon line. Projecting the points 1, 2, 3 and 4 from their respective vanishing points, we get the perspective image $(A)(B)(\Gamma)(\Delta)$ of the figure.

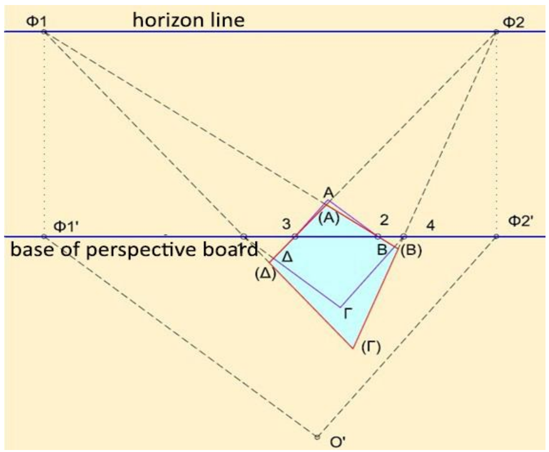

Image 3: Perspective image of the square $AB\Gamma\Delta$ of the horizontal plane E_1

Due to the fact that, using this method of construction, the plan of the object and its perspective images overlap, a parallel shift of the perspective drawing is suggested, as we see in Image 4. Thus we retain the data (base of the perspective board, O' and the plan of the figure) in their original place and we shift in parallel the perspective drawing. The base β of the perspective board shifts in parallel to the original (in a random distance) and the line o of the horizon is placed in parallel to the new position of β, in a determined distance from the data (equal to the height of observation). The point's 1, 2, 3, and 4 (Image 4), on which the sides of the figure $AB\Gamma\Delta$ meet β, are shifted to the new position of β. The vanishing points Φ_1 and Φ_2 arise on the horizon after the projections Φ_1' and Φ_2' have been determined. The projection of the points 1, 2, 3 and 4 from their respective vanishing points results in the perspective image $(A)(B)(\Gamma)(\Delta)$ of the figure.

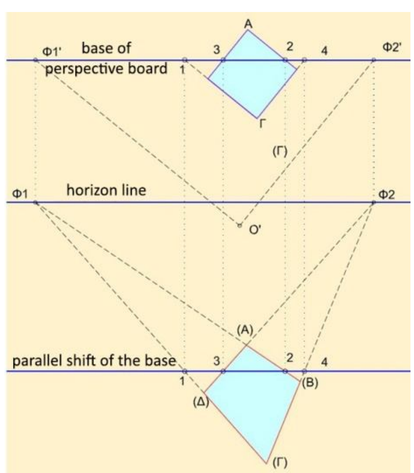

Image 4: Perspective image of the square $AB\Gamma\Delta$ of the horizontal plane E_1

3. Reconstruction from a perspective drawing when an element of the plan of the object is known

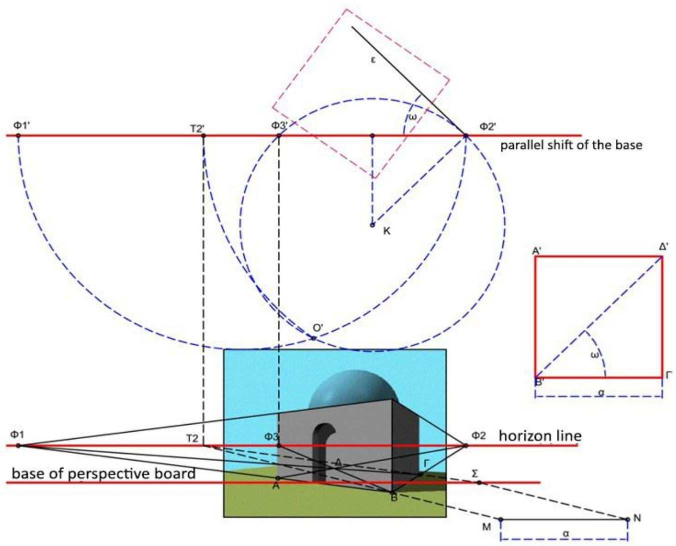

Image 5

It is possible, under certain circumstances, to reconstruct plans and elevations of a building from perspective drawings or photos, using geometrical etchings. We will examine the case where we know some elements of the plan. We assume that during the taking of the photos or the perspective drawing the plane of the perspective board is a vertical plane.

In Image 5 we have a perspective image of a building and at the same time we assume that we know the measurements of the rectangle ΑΒΓΔ. In order to calculate all the measurements, so that a reconstruction of the plan and elevation of the building can be made (as many elements as included in the perspective image), we need to determine the point of view, the height of the horizon line and the position of the perspective board.

Horizon line: In Image 4, the extensions of the parallel horizontal vertexes of the walls intersect in groups of two and determine the vanishing points Φ_1 and Φ_2, from which occurs the horizon line.

Point of view: We assume, in a random position, a parallel shift of the perspective board, which in any occasion will be parallel to the horizon line. If O' is the projection of the point of view O, and Φ_1' and Φ_2' are the projection of the vanishing points Φ_1 and Φ_2 on the parallel shift of the perspective board, then the angle Φ_1'O'Φ_2'=90° (if ΑΒΓΔ is a rectangle). A *locus*, therefore, of O' is the semicircle with diameter Φ_1' Φ_2'. The diagonal ΔΒ of ΑΒΓΔ is a horizontal line and hence has as its vanishing point the point Φ_3 on the horizon line.

If the angle of ΒΓ and ΒΔ is ω, then the angle Φ_2'O'Φ_3'=ω [2]. Consequently the second *locus* of O' is the arc of the circle that faces the segment Φ_2'Φ_3' at an angle ω [3] (Image 5).

[2] This happens because, when we have a given plan and the projection O', the points Φ_1', Φ_2' and Φ_3', which are projections of vanishing points of horizontal lines of the plan, occur on the base of the perspective board from the parallels to O' towards them. Therefore, the angle Φ_1'O'Φ_2' will be equal to the real angle Α'Β'Γ' and the angle Φ_3'O'Φ_2' equal to ΔΒΓ, because they have parallel sides that are in one-to-one correspondence.

Base line (of the perspective board): When O' has been determined, the points of measurement[4] that match the vanishing points Φ_1 and Φ_2 appear instantly. The distance of the point of measurement T_2 from Φ_2 is $T_2\Phi_2 = T_2'\Phi_2' = O'\Phi_2'$. If we project the segment $B\Gamma$ (or $A\Delta$) onto the perspective board, the real length of $B\Gamma = A\Delta = \alpha$ will occur. The perspective board therefore occurs as follows: Initially we project $B\Gamma$ from T_2. Afterwards, we place a line segment $MN=\alpha$ onto a parallel line to the horizon, from a random point M of T_2B. If Σ is the point of intersection of $T_2\Gamma$ and its parallel to T_2B from N, then Σ is a point of the perspective board's base.[5]

4. Incorporation of a perspective drawing in a photo

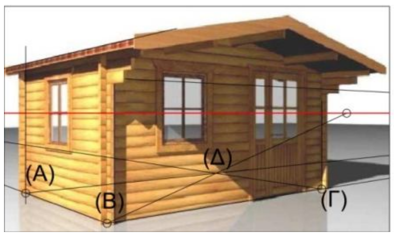

Image 6 Image 7

To incorporate a perspective image of an object that has come about from a photo or a digital modeling, in an image of a given space and at a certain position, the following conditions need to be fulfilled:

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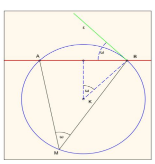
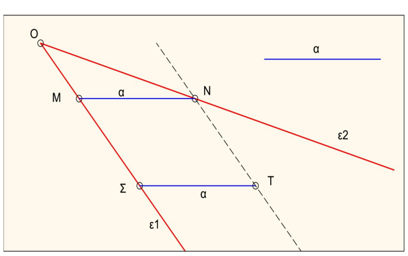
[3]  The segment AB can be seen from any point of the arc AMB from an angle ω. The centre K of this circle is located on the one hand on the perpendicular bisector on AB and on the other hand on the perpendicular of the line ε, which, at point B forms with AB an angle ω. The line ε will be tangent to this circle at point B and the angle that forms from the string AB and the tangent ε will be equal to the inscribed angle AMB. Therefore, the segment AB can be seen from M from an angle ω. If $\omega=90°$, then the arc AMB will be a semicircle.

[4] The point of measurement T1 is a feature point of view on the horizon (and T1' is its projection on the perspective board) that occurs from the arc with centre $\Phi1'$ and radius $O'\Phi1$, O' being the projection of the eye and $\Phi1'$ the projection of a vanishing point on the base of the perspective board. The point T1 has the property that, when it projects a perspective drawing of a line segment, which has as a vanishing point the point $\Phi1$, then on the perspective board occurs the real measurable size of this segment.

[5] 

To put the line segment $MN//=\alpha$ on the angle of the lines ε_1 and ε_2, we assume a random point Σ of ε_1. We construct the segment ΣT equal and parallel to α. The parallel line to ε_1 that comes through T determines the point N on ε_2.

The image that will be incorporated and the image of the space in which the incorporation will take place, need to result from the same data of perspective drawing. This means that the point of view and the axis of view must be the same for both images.

The scale of the perspective drawing, that is a result of the distance of the observer from the perspective board, needs to be the same in both images.

We will examine the two following instances:

In the first instance, which is the more complex, we assume that the image that we will incorporate in the photo of a real space is given and we simply know some of the dimensions of the object that is depicted. Thus, for the wooden house of Image 6, we know the dimensions of the rectangular plan of ΑΒΓΔ. In Image 7 we see a photo of the space where the incorporation will take place.

In this instance, following the process of reconstructing from a perspective drawing that we described earlier, we can determine the perspective elements of Image 6, i.e. the point of view, the perspective board and the horizon (Image 8). When these elements occur, we reconstruct the plan from the perspective drawing and we adjust the position in which we wish to place the object of the image in the real space.

In this particular example we assumed that the wooden house will be placed in contact with the rear limit of the site, therefore the vertex (Α)(Δ) will coincide with the trace of the wall on the ground. We pick the point (Τ') of (Α)(Δ) that is located in the central optical beam and we determine its position on the wall of the site. We assume the vertical segment (T)(Τ'), whose height is equal to the height of the horizon. We materialize this point in the real space of the site, placing for instance a vertical rod on the wall with this specific height. Afterwards, we determine the position of the point of view in the real space of the site, by measuring the distances in the reconstructed plan. In the materialized position of the point of view we place the camera and we aim it to point T. The photo we will take will be of a different scale than the image we wish to incorporate. The element that will allow us to adjust the scale is the dimension TT'. The dimension TT' of the photo of the space and the dimension (T)(Τ') of the image of the wooden house will have to coincide (Image 9).

In the second instance, in which an image that will be incorporated in the real space is not given but exists as an object or as a digital model of some cad software, the reconstruction process is not necessary, because we can choose from the start the position of the point of view to be the same with the position of view axis in both images. What remains in any instance is the problem of scale, which can be solved the way we did in the previous example. We choose a common element of both pictures and we locate its position. This common element may be a vertical dimension TT', preferably on the central optical beam with the same height as the point of view. We materialize this element both in the model and the real space. If we consider the perspective image of the model from the point of view we chose, as well as the image that occurred from the photo we took of the space, we will have two images from the same point of view but in different scale. By adjusting the scale of the two images, we can achieve the correct incorporation of one into the other.

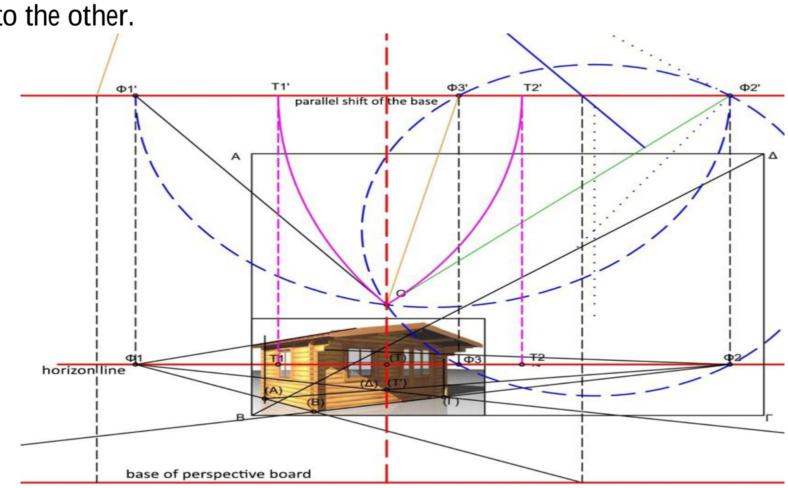


Image 8

Image 9

Afterword

In this paper we discussed the problem of incorporating a perspective drawing in a photo of a space, in such a way as to designate the relation between geometrical perception of space and representation in general. In an architect's training, the visual experience we have of things and the space that surrounds them is often not sufficient for the treatment of the designing problem. Geometry has a role of explaining experience and organizing thought, enhancing the general perception of space.

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