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Minimizing Bending Stresses in Disconnect Pins

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Abstract

Disconnect features on web slings consist of interlocking eye straps through which a round pin is inserted such that the sling may be connected/disconnected at its midsection. These are common features on marine slings, since they make it easier to install basket hitches around large boat hulls. On most slings the width of each finger of the disconnect is uniform on each side. This paper offers guidance on varying the width of the fingers in order to optimize the connection in terms of the bending strength of the pin. Slight changes to the width of the fingers can significantly reduce the maximum bending moment in the pin relative to those caused by uniform finger widths, by a factor of 2.6 for a 2x3 disconnect and by a factor of 4.8 for a 3x4 disconnect. This corresponds to a reduction of the overall weight (cost) of the required pin by 47% and 65%, respectively. A set of equations is also presented to compute ideal finger widths for disconnects of any order, i x (i+1), which minimize bending stress in the pin. It is demonstrated (somewhat surprisingly) that the higher the order of the disconnect, the greater the reduction in maximum bending moment, relative to the moment with uniform finger widths.

Key words: bending stresses, web sling disconnect, disconnect pins, bending stress optimization, shear force distribution, bending moment distribution

1. Introduction

Web slings can be used to lift large objects with surfaces that should be protected from contact with the rough profile of other sling options, such as chains or wire rope. Such slings are often referred to as marine slings, since they are used to lift large boats. Figure 1 shows some slings in action.

Fig. 1 – Marine slings lifting large boats.

To facilitate the lift, such slings can include a disconnect feature enabling the sling to be separated and reconnected along its midsection via a pin and interlocking eye straps. Examples of this disconnect feature are shown in Fig. 2. Note in this paper that the loops that engage the pin are referred to as webbing eyes (consistent with industry terminology), and also as "fingers" since they interlock like fingers on a hand.

As depicted in Fig. 2 that there are two common types of disconnects: 2x3 and 3x4, referring to the number of interlocking eyes (fingers) on each side. Also notice that the width of each finger on either side of the disconnect is equal. As will be discussed, the eye width on the side with fewer fingers is larger than the eyes on the side with more

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fingers. (For example, the three fingers on the 2x3 disconnect are narrower than the two fingers on the other side. But those three fingers have equal widths.)

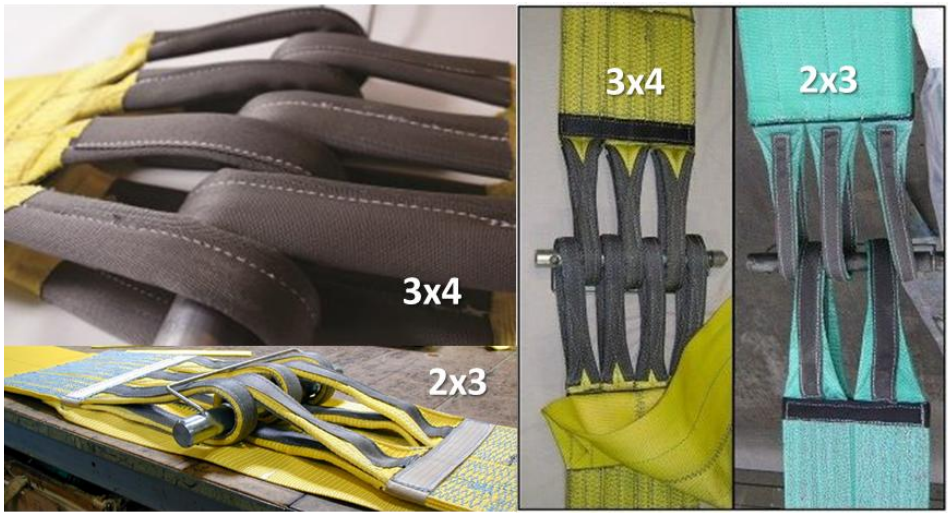

Fig. 2 – Examples of 2x3 and 2x4 disconnect features on web slings.

In this study, the bending moment and corresponding bending stresses in the disconnect pin are analyzed for disconnects with uniform width fingers, using straightforward "mechanics of materials" calculations. It is then demonstrated that these moments/stresses can be reduced substantially by simply slightly changing the widths of the fingers. This results in a significantly less expensive pin to do the same job. Smaller diameter pins also have less potential to damage a boat hull during a lift.

After demonstrating this for the two commonly available 2x3 and 3x4 cases, a general solution is presented for any higher order disconnect, "i x i+1." Even though higher order disconnects are not typically used for a marine sling, this method for joining two flat sheet or plate components is possible. This general solution produced the non-intuitive result that increasing the order of the disconnect, continually decreases the stresses in the pins, relative to the uniform finger width option.

2. Shear Forces and Bending Moments in Disconnect Pins with Uniform Finger Widths

Illustrations of a 2x3 and 3x4 disconnect are shown in Fig. 3. Since most slings are made with fingers of uniform width, a = b for both disconnects in the figure and c = d for the 3x4. The following assumptions are made for this analysis:

1. The rated strength per unit width of the fingers are the same for the webbing eyes on both sides.
2. The fingers on opposing sides of the disconnect are sized such that they carry the same uniform load per unit length, *w*. This happens when the cumulative width of webbing is equal on both sides and symmetrically spaced about the sling centerline.
3. When interlocked, the fingers are separated by a small distance, e, from each other.

The first two assumptions assures that the cumulative strength of all fingers is equal on both sides of the connection. The assumption of a spacing, e, would account for any Poisson-like contraction of the fingers, or incomplete contact with the pin at the edges of the fingers. This is not measured or computed but instead included in the analysis just to assess its influence (which turns out to be small).

<u>For the 2x3 interlock</u> the overall length of the pin, L, is related to the finger geometry by:

$$L - 4e = 2a + b + 2c \quad (1)$$

The second assumption results in the relation:

$$2a + b = 2c \quad (2)$$

This leads to the following equation:

$$c = \frac{L-4e}{4} \quad (3)$$

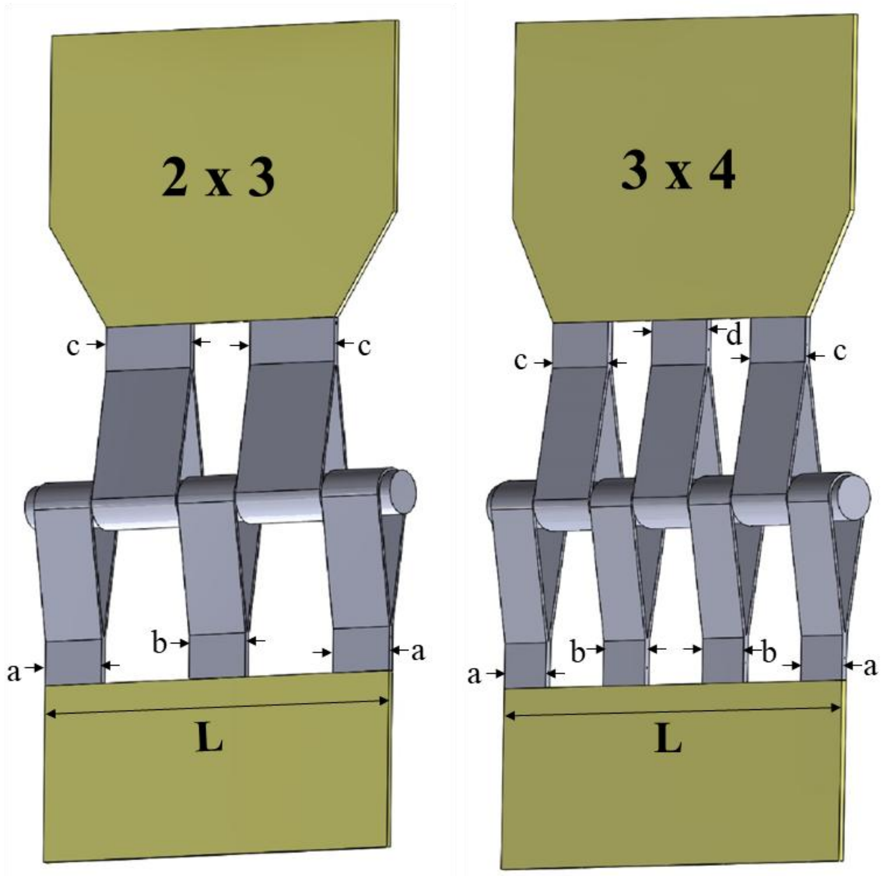

Fig. 3 –Finger widths on typical 2x3 and 3x4 disconnects. Fingers on either side are of uniform width such that a = b for both and d = c for the 3x4.

For the 3x4 interlock:

$$L - 6e = 2a + 2b + 2c + d \quad (4)$$

And the second assumption results in the relations:

$$2a + 2b = 2c + d \quad (5)$$

$$c + \frac{d}{2} = \frac{L-6e}{4} \quad (6)$$

For uniform eye widths on both sides of the sling, $a = b$for the 2x3 in Eqns. (1) and (2) such that

$$c = \frac{3}{2}a \quad (7)$$

$$a = \frac{L-4e}{6} \quad (8)$$

And for the 3x4, $c = d$ and $a = b$ in Eqns. (5) and (6) such that

$$c = \frac{4}{3}a \quad (9)$$

$$a = \frac{L-6e}{8} \quad (10)$$

If the total axial force carried by the sling is F, the distributed load, *w*, exerted by the webbing on each side of the 2x3 pin is computed by:

$$w = \frac{2F}{L-4e} \tag{11}$$

and the distributed load for the 3x4 pin is

$$w = \frac{2F}{L-6e} \tag{12}$$

For fingers of uniform width (a = b and c = d), the distributed loading and corresponding shear force and bending moment diagrams for the 2x3 and 2x4 disconnect pins are shown in Fig. 4 for an example case with a force of F = 100 kips, L = 12 in, and e = 0.06 in.

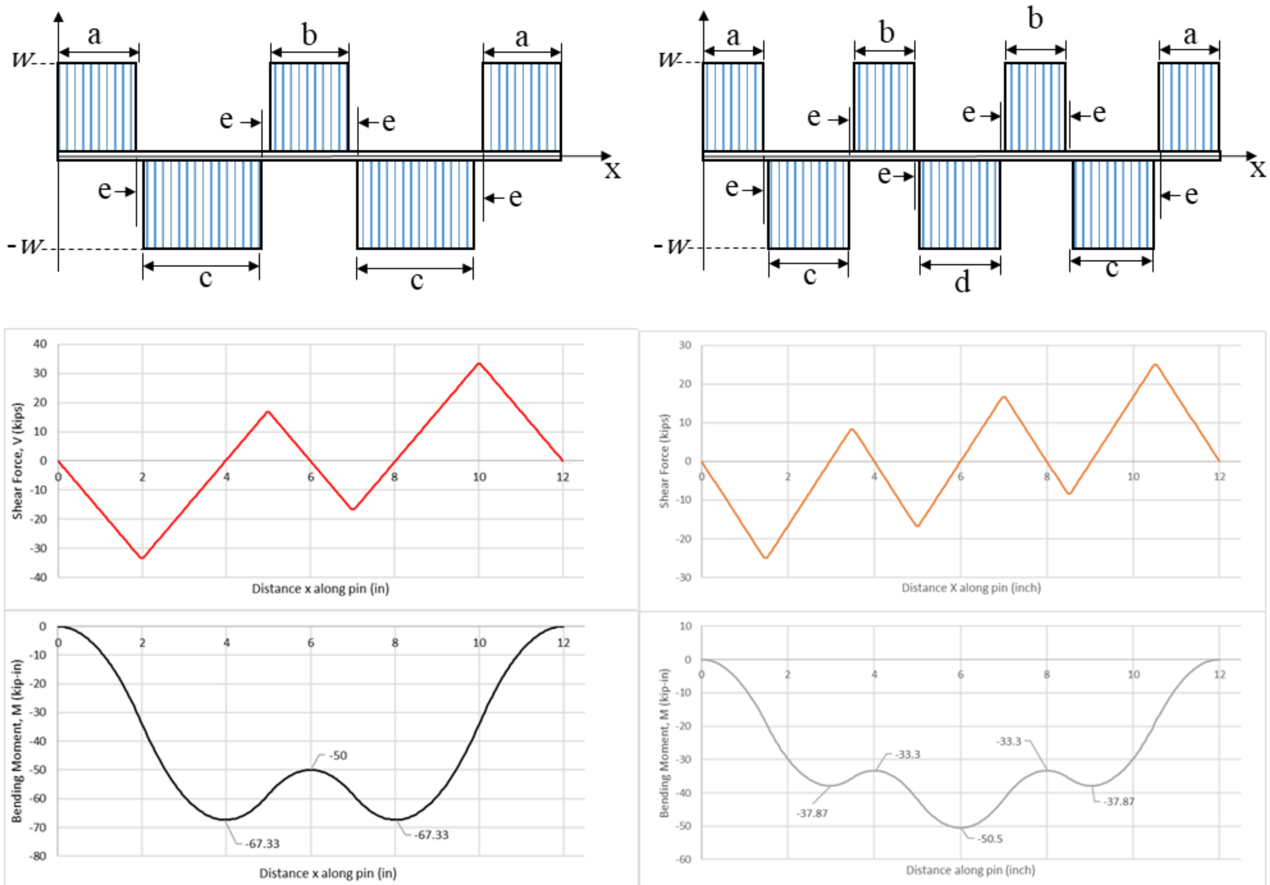

Fig. 4 – Distributed loading and corresponding shear and moment diagrams for 2x3 and 3x4 pins with uniform finger widths. This is for an example case with L = 12 inch, e = 0.06 inch, and F = 100 kips.

The shear and moment diagrams can be generated by making analytical sections some distance, x, along the pin, then solving for the internal shear and moments that keep the pin in static equilibrium [1]. Alternatively, the distributed loads may be integrated along the length of the pin, x, to generate the shear diagram and the shear diagram may be integrated along its length to produce the moment diagram. In this uniform eye width example, the sling and loading parameters are given in Table 1, along with results for both uniform and optimized finger widths, to be described.

Notice in Table 1, the diameter of a pin is also computed that would have a factor of $n_y = 2$ against yielding, if the material has a yield strength of 40 ksi. (i.e., the diameter such that the bending stress caused by M_{max} is 20 ksi).

Table 1 - Summary for 12-inch Sling Example Geometry*					
	2 x 3			3 x 4	
	Uniform	Optimal		Uniform	Optimal
a (in)	1.96	1.21	a (in)	1.46	0.75
b (in)	1.96	3.45	b (in)	1.46	2.16
c (in)	2.94	2.94	c (in)	1.94	1.83
			d (in)	1.94	2.16
w (kip/in)	17.01	17.01	w (kip/in)	17.18	17.18
M_{max} (kip-in)	67.33	26.25	M_{max} (kip-in)	50.5	10.23
diameter (in)	3.249	2.373	diameter (in)	2.952	1.734
% reduction in dia	26.90%		% reduction in dia	41.30%	
% reduction in cost	46.60%		% reduction in cost	65.50%	
* F = 100 kips, L = 12 in, e = 0.06 in, S_y = 40 ksi, n_y = 2.0					

Notice from Fig. 4 in both cases that the bending moment is entirely negative along the full length of the pins. The maximum magnitude of the moment (M_{max}) is 67.33 kip in for the 2x3 case, occurring at the two local minimums in the moment diagram (corresponding to the location where the shear stress is zero). For the 3x4 case, the maximum moment magnitude is50.3 kip in and occurs at the middle of the pin.

3. Shear Forces and Bending Moments in Disconnect Pins with Optimal Finger Widths

The fact that the moments are fully negative along the entire length of the pin suggests that the distributed loading could be modified by changing the width of the fingers to cause the peaks and valleys of the moment distribution to lie above and below zero, respectively, in both cases. This would serve to reduce the magnitude of the maximum bending moment. In fact, a set of finger width may be computed to cause the peaks and valleys to have equal magnitudes everywhere along the pin. This situation represents the minimum achievable bending moment magnitude in the pin. This optimization philosophy is described in references such as [1-3] for beam design problems.

The distributed loading and corresponding shear and moment diagrams for optimal finger width scenarios are presented in Fig. 5 for both the 2x3 and 3x4 cases. The dimensions that cause the optimal bending moments can be found iteratively using standard “mechanics” solutions described already. These optimal dimensions are presented in Table 1 and the corresponding distributed loads are shown to scale in Fig. 5 for 2x3 and 3x4 disconnect pins, along with the corresponding shear and moment diagrams. Inspection of these diagrams helps explain how closed form equations may be derived to compute the values of “a” and “b” that cause peaks and valleys of the moment diagrams to have equal magnitudes along the length of the pin in each case, thus assuring the minimum possible bending moment.

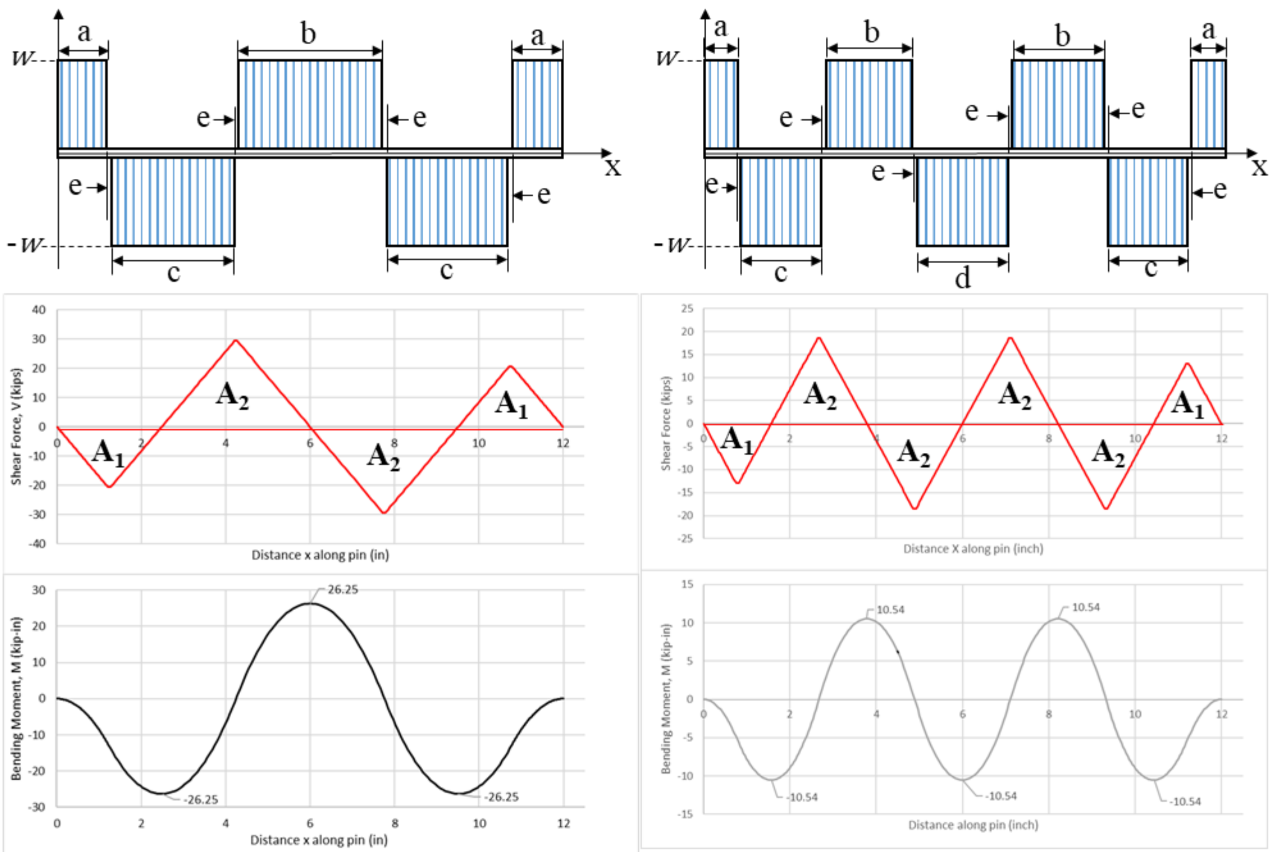

Fig. 5 – Distributed loading and corresponding shear and moment diagrams for 2x3 and 3x4 pins with optimal finger widths.

Recall that the distributed loading, *w*, has an equal value on each side of the pin. This is achieved by making the cumulative (gross) width of all fingers equal on each side of the disconnect. This is done to make sure that the fingers have the same strength on each side of the connection, which makes the slopes of the shear diagram equal and opposite along x such that the areas A_1 and A_2 are isosceles trapezoids. (Unless e = 0, then they are isosceles triangles.)

As we move along the length, x, of the shear diagram, the areas A_1 and A_2 are summed to obtain the peaks and valleys of the moment diagram. Therefore the first positive area A_2 must be double the negative area A_1 in order for the first peak moment to equal the first valley moment. This is true for both the 2x3 and 3x4 cases.

Refer to Fig. 6 for a closer look at the 2x3 shear diagram. Since the slope of all angled segments is equal and opposite, the area A_1 must be given by:

$$A_1 = 2\left(\frac{1}{2}(wa)a\right) + (wa)e = wa^2 + wae \quad (13)$$

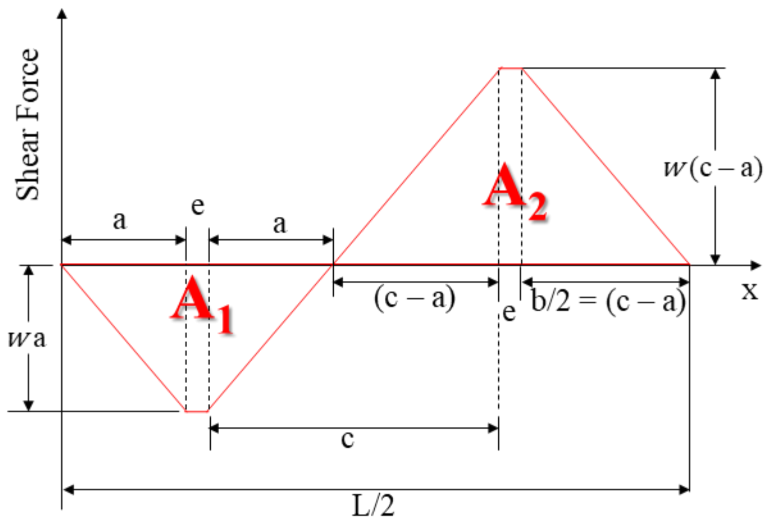

Fig. 6 – Shear diagram for $0 \leq x \leq L/2$ in the 2x3 pin. The negative area A_1 represents the moment at x = 2a + eand (A_1 + A_2) is the moment at x = L/2= a + c + b/2 + 2e.

Referring to Figs. 5 and 6 for the 2x3 disconnect, it is apparent that b/2 must equal (c-a). Therefore, for the moment at L/2 to be equal and opposite the moment A_1 at x = 2a + e, A_2 must equal $2A_1$:

$$A_2 = 2A_1 \qquad (14)$$

Or:

$$w(c-a)^2 + w(c-a)e = 2(wa^2 + wae) \qquad (15)$$

(Remember that A_1 and A_2 refer to the magnitude of the trapezoidal areas. In Fig. 6, A_1 is negative and A_2 is positive.) Omitting algebra, Eqn. (15) may be combined with Eqn.(3) to and solved to compute the following equation for *a*:

$$a = \sqrt{\left(\frac{L^2}{8} + \frac{e^2}{4}\right)} - \frac{1}{2}\left(\frac{L}{2} + e\right) \qquad (16)$$

With *a* from Eqn. (16), and *c* being known from Eqn. (3) and Eqn. 2 is used to obtain Eqn. (17) for *b*:

$$b = 2(c-a) \qquad (17)$$

This results in the dimensions presented in Table 1 and moment diagram in Fig. 5 where the maximum moment for the 12-inch long 2x3 pin is reduced from 67.3 kip-in to 26.25 kip-in (a factor of 2.57 reduction in moment!) Note that the pin diameter required for a safety factor of 2 is reduced from 3.25 inches down to 2.37 inches, representing a weight (cost) reduction of 47%.

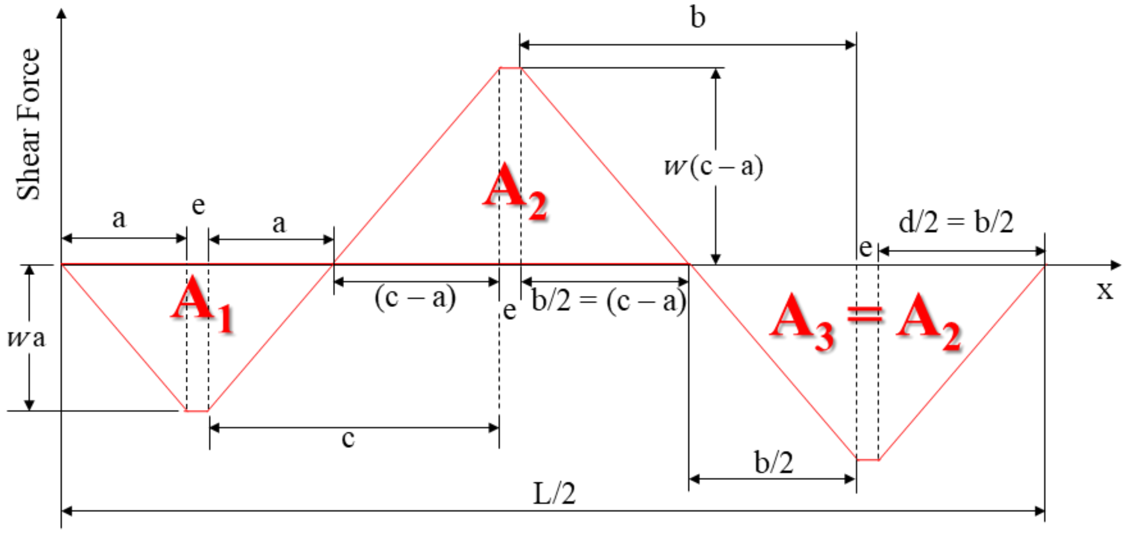

Fig. 7 - Shear diagram for $0 \leq x \leq L/2$ in the3x4pin. Area A_1 represents the moment at x = 2a + e, $A_1 + A_2$ is the moment at x = a + c + b/2 + 2e, and $A_1 + A_2 + A_3$is the moment at x = L/2 = a + c + b + d/2 + 3e.

Now for the 3x4 pin, looking at the shear force diagram in Fig. 7, A_2 again must equal twice the magnitude of A_1 to cause equal and opposite moments at local minima and maxima points(x = 2a + e, and x = a+c+b/2 +2e, respectively). In this case c cannot be computed directly, but in order to cause an equal and opposite moment at x = L/2, it is apparent that A_3 must be equal in magnitude to A_2. Since the slopes of the shear diagram are all equal and opposite, this means that d must be equal to b.

Therefore, since *d* = *b*, then an expression for *a* is given by:

$$a = \frac{L-6e-4b}{4} = \left(\frac{L-6e}{4}\right) - b \quad (18)$$

or:

$$b = \left(\frac{L-6e}{4}\right) - a \quad (19)$$

Therefore, the expression

$$A_2 = 2A_1 \quad (20)$$

Is now given by:

$$w\left(\frac{b^2}{4} + \frac{be}{2}\right) = 2(wa^2 + wae) \quad (21)$$

or

$$\frac{b}{2}\left(\frac{b}{2} + e\right) = 2(a^2 + ae) \quad (22)$$

By substituting Eqn. (19) into (22) we have an equation that may be solved for *a*:

$$\frac{1}{2}\left(\left(\frac{L-6e}{4}\right) - a\right)\left(\frac{\left(\frac{L-6e}{4}\right)-a}{2} + e\right) = 2(a^2 + ae) \quad (23)$$

Omitting algebra, Eqns. (23) is usedto compute the following equation for *a*:

$$a = \sqrt{\frac{1}{7}\left(\frac{L^2}{14} + e^2\right)} - \frac{1}{2}\left(\frac{L}{14} + e\right) \quad (24)$$

With *a* known, *b* is computed from Eqn. (19), and *d*= *b*, and c is computed from Eqn. (6) to be:

$$c = \frac{L-6e}{4} - \frac{b}{2} \tag{25}$$

For the example geometry (L =12 in. and e = 0.06 in.) the dimensions presented in Table 1 are computed from these equations (Eqns. (24), (19) and (25)), and were used to make the shear and moment diagrams in Fig. 5 for the optimized 3x4 disconnect (a = 0.754 in., b = 2.156 in., c = 1.832 in. and d = b = 2.156 in). As seen in Table 1 and Fig. 5, the minimum bending moment is reduced from 50.5 kip-in down to 10.3 kip-in (or by a factor of 4.8!). Note that the required pin diameter in this case drops from 2.96 inch, down to 1.74 inch, representing a 66% reduction in weight (cost).

4. Summary, Discussion and Derivation of Generalized Equation

As noted previously, the smaller maximum moment in the pins justifies the use of a smaller diameter pin to achieve a desired safety factor for a specific material strength. A moment of M results in a maximum bending stress in the pin of diameter D equal to:

$$\sigma_{max} = \frac{32M}{\pi D^3}$$

For a pin of yield strength S_y to be safe from yielding by a factor of n_y, then its maximum stress should be less than the yield strength by:

$$\sigma_{max} = \frac{S_y}{n_y}$$

If the bending moment in a pin of diameter D_1 with uniform finger spacing is M_1, corresponding to a stress of σ_{max}, and the moment is reduced to M_2 by optimal finger spacing, the optimal pin diameter D_2 is related to D_1 by

$$\frac{D_2}{D_1} = \left(\frac{M_2}{M_1}\right)^{1/3} \tag{26}$$

And the cost of the pin is directly proportional to its weight, or cross sectional area, such that the area ratio of the smaller to larger pins is given by:

$$\frac{A_2}{A_1} = \left(\frac{M_2}{M_1}\right)^{2/3} \tag{27}$$

Figure 8 plots the diameter and area ratios using Eqns. (26) and (27) to show the influence of *e* over a range of 0 to 0.55 inches with an overall width of 12 inches.

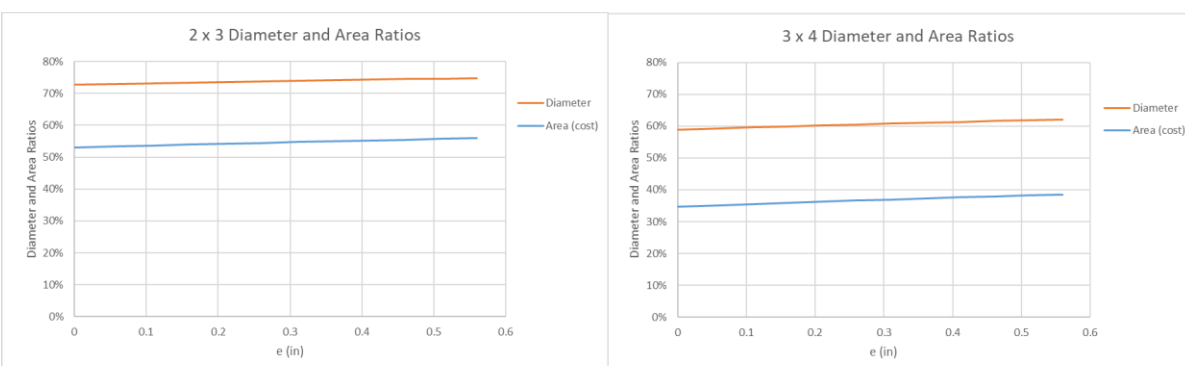

Fig. 8 – Ratio of diameters and area (cost) for pins in optimized versus uniform finger spacing for a 12 inch pin length used in2X3 and 3X4 disconnects.

Figure 8 shows that the effect of e is minor up to a gap of ½ inch on a 12 inch width, and that a greater diameter and cost reduction is achieved in the 3x4 disconnect than in the 2x3.

Table 1 provides a summary for the 2x3 and 2x3 disconnect pin example cases. This table includes the maximum bending moment and stress comparison for disconnects with uniform and optimal finger widths. It also presents the minimum pin diameters based on a yield strength of 40 ksi and a safety factor of 2.0.

Note that the cost of the 3x4 pin is reduced by 65.5% for the optimal width disconnect, as the minimum diameter drops from close to 3 inches down to 1.75 inch. The required 2x3 pin drops from a diameter of about 3.25 inch down to 2.37 inch, a cost reduction of 47%. The difference in required pin size from a uniform 2x3 to an optimized 3x4 is even greater: a 71.5% cost reduction is achieved by reducing the maximum bending moment from 67.3 kip-in down to 10.23 kip-in.

The two most important equations in this paper are Eqns. (16) and (24) for the width *a* in the 2x3 and 3x4 disconnect, respectively. With these two values, the corresponding maximum bending moment is

$$M_{max} = wa(a + e) \tag{28}$$

It should be noted that the maximum lateral bending deflection of the pin with the optimal finger widths for the 3x4 test case was on the order of 0.0012 inch for aluminum pins, reduced by about a factor of 8 relative to the uniform widths. Similar lateral bending deflections were computed for the 2x3 pin (about 0.002 inch for optimal widths, reduced by about a factor of 8 from the pin with uniform finger widths). Compared to the compliance of the webbing, these deflections are negligible, supporting the assumption of a uniformly distributed load.

Looking back at the 3x4 case in Fig. 7, it is clear that each trapezoid inward of A_2 must also have an area equal to $2A_1$ along the entire shear diagram, such that the moment fluctuates between $-A_1$ and $+A_1$. Therefore, for higher order disconnects {e.g, i x (i+1), where i > 3} all internal fingers must have the same width of b. Using this fact with geometry and algebra, the following general relation for *a* was derived for an i x (i+1) disconnect, with a length of L and a gap between the fingers of e:

$$a = \sqrt{\frac{(i-1)^2}{4}\left(\frac{L^2}{2(2i^2-4i+1)^2} + \frac{e^2}{(2i^2-4i+1)}\right)} - \frac{1}{2}\left(\frac{L}{2(2i^2-4i+1)} + e\right) \tag{29}$$

and

$$b = \frac{(i-1)L}{(2i^2-4i+1)} - e - \sqrt{\frac{L^2}{2(2i^2-4i+1)^2} + \frac{e^2}{(2i^2-4i+1)}} \tag{30}$$

and

$$c = a + \frac{b}{2} \tag{31}$$

Note that Eqn. (29) is identical to Eqn. (16) when i =2 (2x3), and identical to Eqn. (24) when i = 3 (3x4)

To summarize, Equations (29)-(31) provide the dimensions for any i x (i+1) disconnect that induce the minimal bending moment on the pin. The moment for these finger widths may be computed by Eqn. (28), (repeated below)

$$M_{max} = wa(a + e) \tag{28}$$

along with Eqn. (32) below to compute *w*:

$$w = \frac{2F}{L-2ie} \tag{32}$$

These equations were used to compute the example dimensions and moments presented in Table 2 for a 12 inch wide pin with e = 0.06 inch with a sling force of 100 kips.

Table 2 - Higher Order Disconnects								
12	L (in)							
0.06	e (in)					Max Moments		
100	F (kip)		Optimal finger widths			Optimal	Uniform	Moment
i	order	w (kip/in)	a (in)	b (in)	c (in)	M (kip-in)	M (kip-in)	Reduction factor
2	(2x3)	17.01	1.213	3.455	2.940	26.25	67.33	2.56
3	(3x4)	17.18	0.754	2.156	1.832	10.54	50.50	4.79
4	(4x5)	17.36	0.543	1.558	1.322	5.68	36.36	6.41
5	(5x6)	17.54	0.421	1.214	1.028	3.55	30.30	8.53
6	(6x7)	17.73	0.342	0.991	0.838	2.44	24.73	10.13
7	(7x8)	17.92	0.287	0.834	0.704	1.78	21.64	12.13
8	(8x9)	18.12	0.246	0.718	0.605	1.36	18.70	13.72
9	(9x10)	18.32	0.214	0.629	0.529	1.08	16.83	15.61
10	(10x11)	18.52	0.189	0.558	0.468	0.88	15.02	17.16
20	(20x21)	20.83	0.080	0.244	0.202	0.23	7.56	32.62
30	(30x31)	23.81	0.044	0.142	0.115	0.11	5.04	45.68
50	(50x51)	33.33	0.018	0.061	0.048	0.05	3.03	66.56

Notice in Table 2 that the influence of optimal vs uniform finger spacing increases with higher order disconnects (many fingers). This is counterintuitive since it seems logical to expect that for a very large number of fingers, the width of the two outer fingers would have little or no influence on the bending moment along the entire length of the pin. But as shown in Table 2, as the ratio of uniform to optimal bending moment (Moment reduction) increases with i. However it should be noted that the values for a, b and c become unrealistically small for very large numbers, such that assumptions regarding the uniformly distributed load, *w*, are potentially no longer valid for a 12-inch overall pin length (sling width).

Figure 9 shows moment diagrams for the first five cases in Table 2 (i = 2 to 6), to illustrate how the moment is minimized by this approach from the uniform width fingers to the optimal finger widths.

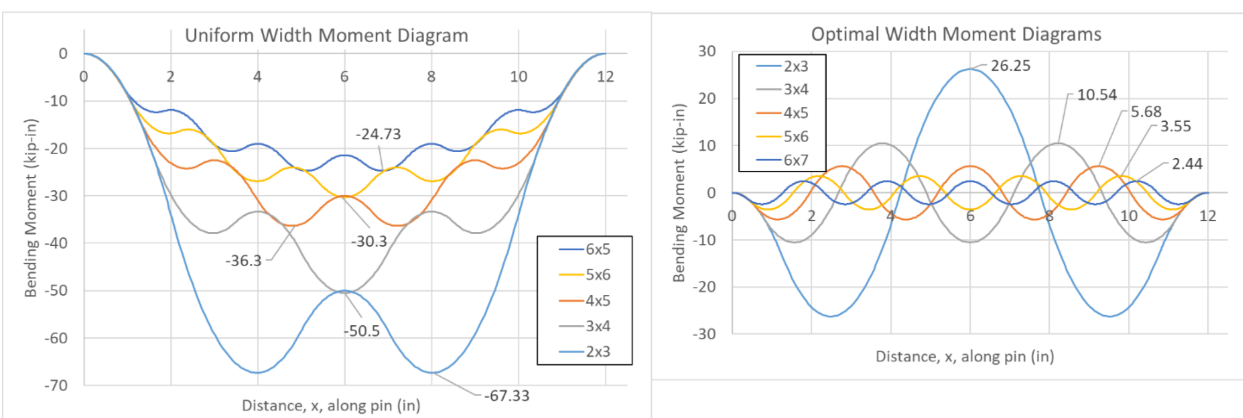

Fig. 9 – Moment diagrams for pins with uniform and optimal finger widths for the first 5 cases in Table 2.

Finally, to end on a practical note in terms of marine slings, there are no known marine slings commercially available with an order higher than 3x4, and more 2x3 disconnects are available than 3x4's. If e is set to a value of zero, then Equations 16 and 24 reduce to

$$a = \left(\sqrt{\frac{1}{8}} - \frac{1}{4}\right) L = 0.10355\ L \tag{33}$$

for the 2x3 and

$$a = \sqrt{\frac{1}{7}\left(\frac{1}{14}\right)} - \frac{1}{2}\left(\frac{L}{14}\right) = 0.0653\ L \tag{34}$$

for the 3x4, respectively. With these values of a the other finger widths are easily defined as constants times the sling width L. Similarly, the maximum bending moment in the pins may be defined as constants times FL, where F is the total force applied to the disconnect. Tables are presented in the Appendix with these constants for 2x3 and 3x4 disconnects.

5. Conclusions

The bending moment and stresses caused by interlocking fingers on the pins in marine sling disconnects are computed for 2x3 and 3x4 disconnects with uniform finger widths on each side of the pin. The moment diagrams for uniform widths are fully negative along the entire length of the pin. However, adjusting the widths of the fingers, the maximum bending moments are minimized by causing local maximum/minimum moments to be equal and opposite along the length of the pin. Equations(16) and (24) are presented for the 2x3 and 3x4 cases, respectively, to compute the optimal finger width geometry. It is demonstrated that significant (e.g. 65%) cost savings can be achieved for pins with the optimally sized fingers, relative to pins for uniformly sized fingers. Equations (29)-(31) are presented to compute optimal interlocking finger widths that minimize bending stress in the pin fora disconnect of any size, i x (i+1) (where $i \geq 2$). Equation (29) is identical to Eqs. (16) and (24) for i = 2 and 3, respectively. These equations reveal that the higher the order of the disconnect (value of i), the greater the bending moment reduction in the pin, relative to the moment caused by uniform finger widths.

6. References

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Appendix

Setting e = 0 in Equations 16 and 24, results in the following tables for finger widths and maximum bending moments, shown with the same constants for uniform finger widths in 2x3 and 3x4 disconnects, respectively.

2 x 3 Disconnect with e = 0	
uniform finger widths	optimal finger widths
$a = \frac{L}{6} = 0.16667\ L$	$a = \frac{L}{4}\left(\sqrt{2} - 1\right) = 0.10355\ L$
$b = \frac{L}{6} = 0.16667\ L$	$b = \frac{L}{2}\left(2 - \sqrt{2}\right) = 0.29289\ L$
$c = \frac{L}{4} = 0.25\ L$	$c = \frac{L}{4} = 0.25\ L$
$M_{max} = \frac{FL}{18} = 0.05556\ FL$	$M_{max} = FL\left(\frac{2 - \sqrt{2}}{4}\right)^2 = 0.021447\ FL$

3 x 4 Disconnect with e = 0	
uniform finger widths	optimal finger widths
$a = \frac{L}{8} = 0.125\ L$	$a = \frac{L}{7}\left(\frac{1}{\sqrt{2}} - \frac{1}{4}\right) = 0.06530\ L$
$b = \frac{L}{8} = 0.125\ L$	$b = \frac{L}{7}\left(2 - \frac{1}{\sqrt{2}}\right) = 0.18470\ L$
$c = \frac{L}{6} = 0.16667\ L$	$c = \frac{L}{28}\left(3 + \sqrt{2}\right) = 0.15765\ L$
$d = \frac{L}{6} = 0.16667\ L$	$d = b = 0.18470\ L$
$M_{max} = \frac{FL}{24} = 0.04167\ FL$	$M_{max} = FL\left(\frac{4 - \sqrt{2}}{7\sqrt{2}}\right)^2 = 0.008528\ FL$